Intuitionistic Fuzzy Soft Function And It's Application In The Climate System

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Abstract: The main purpose of this paper is to introduce the concept of intuitionistic fuzzy soft function (IFS-function). We have defined here different types of IFS-functions with appropriate examples. Moreover we have proven some theorems on these IFS-functions. Finally we have applied IFS-functions and their operations to solve a problem regarding one of the most happening subject- the Massive Change of Climate in current context.

Keywords: Intuitionistic Fuzzy Soft Function (IFS-function), composition, inverse, application.

I Introduction
To deal with uncertainties in our real life situations we need strategies which provide some flexible information processing capacity. Soft set theory is generally used to solve such problems. Initially Molodtsov [3] presented soft set as a completely generic mathematical tool for modeling uncertainties in the year 1999. Since there is hardly any limitation in describing the objects, researchers simplify the decision making process by selecting the form of parameters they require and subsequently makes it more efficient in the absence of partial information. Maji et al. [8,9,10] have done further research on soft set theory and on fuzzy soft set theory in 2001, 2002 and 2003. To continue the investigation on fuzzy soft sets, Kharal et al.[1] introduced the concept of a mapping on the classes of fuzzy soft sets. 

From the discussion above, we can see that all of these works are based on the classical soft set theory. The soft set model, however, can also be combined with other mathematical models. For example, by amalgamating the soft set and algebra, Aktas et al.[5] proposed the definition of soft groups, Feng et al. [4] proposed the concept of soft semirings, soft subsemirings, soft ideals, idealistic soft semirings and soft semiring homomorphisms. Then in the year 2009, Yang et al. [13] have combined the interval-valued fuzzy set [7] and soft set[3], from which a new soft set model: interval-valued fuzzy soft set[13] is obtained. In the year 2010, Babitha et al.[6] have introduced the concept of soft set function. Presence of vagueness demanded Fuzzy Soft Set (FSS) [10] to come into picture, but satisfactory evaluation of membership values is not always possible because of the insufficiency in the available information (besides the presence of vagueness).

Evaluation of non-membership values is also not always possible for the same reason and as a result there exists an indeterministic part upon which hesitation survives. Certainly fuzzy soft set theory is not suitable to solve such problems. In those situations Intuitionistic Fuzzy Soft Set theory (IFSS)[11, 12] may be more applicable. In the year 2010 Dinda et al.[2] have worked on intuitionistic fuzzy soft relation.

In this paper first we have proposed the concept of intuitionistic fuzzy soft function(IFS-function). Here we have defined different types of IFS-function and also discussed them by appropriate examples. Moreover we have proven some theorems on these IFS-functions. Finally we have applied IFS-functions and their operations to solve a problem of evaluating the rate of harmful effects on climate due to human activities.

II Preliminaries
A Definition: Soft Set [3]
Let $U$ be an initial universe set and $E$ be a set of parameters. Let $P(U)$ denotes the set of all subsets of $U$. Let $A \subseteq E$. Then a pair $(F, A)$ is called a soft set over $U$, where $F$ is a mapping given by, $F : A \rightarrow P(U)$.

In other words, a soft set over $U$ is a parameterized family of subsets of the universe $U$.

B Definition: Fuzzy Soft Set (FSS) [10]
Let $U$ be an initial universe set and $E$ be a set of parameters(which are fuzzy words or sentences involving fuzzy words). Let $F^U$ denotes the set of all fuzzy subsets of $U$. Let $A \subseteq E$. Then a pair $(F, A)$ is called a Fuzzy Soft Set (FSS) over $U$, where $F$ is a mapping given by, $F : A \rightarrow F^U$.

C Definition: Intuitionistic Fuzzy Soft Set (IFSS) [11]
Let $U$ be an initial universe set and $E$ be a set of parameters and $IF^U$ denotes the collection of all intuitionistic fuzzy subsets of $U$. Let $A \subseteq E$. Then a pair $(F, A)$ is called an Intuitionistic Fuzzy Soft Set(IFSS) over $U$, where $F$ is a
mapping given by, \( F : A \rightarrow I F^U \)

An intuitionistic fuzzy soft set \((F, A)\) can be viewed as a collection of approximations like below (considering \( A \) having \( n \) members)

\[
(F, A) = \{ p_1 = v_1, p_2 = v_2, \ldots, p_n = v_n \}
\]

where each approximation has two parts viz.,

i) a predicate \( (p_i) \) and

ii) an approximate value-set \( (v_i) \), or simply called value-set \( (v_i) \).

III The Concept of IFS-Functions

A Definition: Intuitionistic Fuzzy Soft Function

Let \((F, A)\) and \((G, B)\) be two non-empty intuitionistic fuzzy soft sets over the same universe \( U \). Then an intuitionistic fuzzy soft relation \( f \) between \((F, A)\) and \((G, B)\) is called an intuitionistic fuzzy soft function (IFS-function) or, intuitionistic fuzzy soft mapping or, intuitionistic fuzzy soft transformation from \((F, A)\) to \((G, B)\) if each element of \((F, A)\) is related to a unique element of \((G, B)\) with a supporting non-null IFS by a rule \( f \) and it is symbolically denoted as \( f : (F, A) \rightarrow (G, B) \).

Here \((F, A)\) is said to be the domain of \( f \) (which is denoted by \( D_f \)) and \((G, B)\) is called the co-domain of \( f \).

Mathematical Representation of an IFS-function

Let \((F, A)\) and \((G, B)\) be two non-empty intuitionistic fuzzy soft sets over the same universe \( U \). Now if \( f \) is an IFS-function from \((F, A)\) to \((G, B)\), then \( f \subseteq (F, A) \times (G, B) \), i.e., \( f \) is of the form \((H, C)\) where \( C \subseteq A \times B \) and \( H : C \rightarrow I F^U \) s.t.,

\[
H(a, b) = F(a) \cap G(b) \forall (a, b) \in C
\]

with the condition \( H(a, b_j) = H(a, b_k) \Rightarrow j = k \). In the parlance of IFS-function we denote \( H(a, b) = \{ u((\mu(u), \nu(u)) : \forall u \in U \} \) as \( F(a) = G(b) \) with the supporting Intuitionistic Fuzzy Set\( (IFS) \) \( \{ u((\mu(u), \nu(u)) : \forall u \in U \} \); [where \( \mu(u) = \min \{ \mu_{F(a)}, \mu_{G(b)} \}, \nu(u) = \max \{ \nu_{F(a)}, \nu_{G(b)} \} \); \( \mu(x), \nu(x) \) respectively denote the membership and non-membership value of \( x \).]

Thus the mathematical representation of an IFS-function \( f : (F, A) \rightarrow (G, B) \) is defined as, \( f(F(a)) = G(b) \) with a non-null supporting IFS; \( F(a) \in (F, A), G(b) \in (G, B) \)

Property:

Let \((F, A)\) and \((G, B)\) be two non-empty intuitionistic fuzzy soft sets over the same universe \( U \). Now if \( f \) is an IFS-function from \((F, A)\) to \((G, B)\), then

\[
f(F(a_i)) = G(b_j) \text{ with a non-null IFS,}
\]

\[
f(F(a_i)) = G(b_k) \text{ with a non-null IFS}
\]

\[
\Rightarrow b_j = b_k.
\]

Example 3.1

Suppose that the set of universe \( U \) contains five students \( s_1, s_2, s_3, s_4, s_5 \) just passed the exam of 10th class. Now they have several ways to travel their future and this is the subject of our example. Let the set of parameters \( E = \{ M.Sc, M.A, M.Com, B.Tech, Researcher, Engineer, Chattered, Accountant, Bank, Research Institute, Hospital, Multi National Company(MNC) \} \)

Let \( A = \{ M.Sc, M.A, M.Com, B.Tech \} = \{ a_1, a_2, a_3, a_4 \} \)

and \( B = \{ Researcher, Engineer, Chattered Accountant, Doctor, Bank, Research Institute, Hospital, Multi National Company(MNC) \} \)

Now let \((F, A)\) and \((G, B)\) be two IFSS respectively describing the academic qualifications the students may take and the professions the students may have are respectively given by,

\[
(F, A) = \{ F(a_1) = \{ s_1/(8,1), s_2/(1,7), s_3/(5,5), s_4/(2,7), s_5/(5,5) \},
\]

\[
F(a_2) = \{ s_1/(1,8), s_2/(9,1), s_3/(5,4), s_4/(7,2), s_5/(5,5) \},
\]

\[
F(a_3) = \{ s_1/(4,5), s_2/(6,3), s_3/(5,5), s_4/(2,7), s_5/(5,3) \},
\]

\[
F(a_4) = \{ s_1/(9,1), s_2/(1,7), s_3/(7,1), s_4/(3,7), s_5/(4,5) \}
\]

\[
(G, B) = \{ G(b_1) = \{ s_1/(8,1), s_2/(7,2), s_3/(3,5), s_4/(6,3), s_5/(4,4) \},
\]

\[
G(b_2) = \{ s_1/(9,1), s_2/(1,7), s_3/(7,1), s_4/(3,7), s_5/(4,5) \},
\]

\[
G(b_3) = \{ s_1/(4,5), s_2/(6,3), s_3/(5,5), s_4/(2,7), s_5/(5,3) \},
\]

\[
G(b_4) = \{ s_1/(2,7), s_2/(7,2), s_3/(5,2), s_4/(2,7), s_5/(3,4) \}
\]

Now let \( f \) describing suitable professions is defined by \( f : (F, A) \rightarrow (G, B) \) s.t.,

\[
f(F(a_i)) = G(b_i) \text{ with supporting IFS}
\]

\[
\{ s_1/(8,1), s_2/(1,7), s_3/(3,5), s_4/(2,7), s_5/(4,4) \}
\]
Now let $F(a_1) 
eq F(a_2)$ implies $f(F(a_1)) 
eq f(F(a_2))$; $F(a_1), F(a_2) \in (F, A)$

C Definition: Injective IFS-function

An IFS-function $f$ from $(F, A)$ to $(G, B)$ is called injective if

$$f(F(a_1)) 
eq f(F(a_2)) \Rightarrow \text{ either } G(b_1) \neq G(b_2) \text{ or } I_1 \neq I_2.$$

Example 3.3

Consider the example 3.1

Now let $A = \{a_1, a_2, a_3, a_4\}$ and $B = \{b_1, b_2, b_3, b_4\}$. Then $(F, A)$ and $(G, B)$ be two IFSS. Now let $h: (F, A) \rightarrow (G, B)$ s.t.,

$$h(F(a_1)) = G(b_1) \text{ with supporting IFS } \{s_1/(.8, 1), s_2/(.1, 7), s_3/(.3, 5), s_4/(.2, 7), s_5/(.4, 4)\}$$

$$h(F(a_2)) = G(b_2) \text{ with supporting IFS } \{s_1/(.4, 5), s_2/(.6, 3), s_3/(.5, 5), s_4/(.2, 7), s_5/(.5, 3)\}$$

Since $F(a) \neq F(b)$ implies $h(F(a)) \neq h(F(b))$; $a \in A, b \in B$

So $h$ is an injective IFS-function.

D Definition: Surjective IFS-function

An IFS-function $f$ from $(F, A)$ to $(G, B)$ is called surjective if for each element $G(b_1)$ of $(G, B)$, $\exists$ atleast one element $F(a_j)$ in $(F, A)$ s.t., $f(F(a_j)) = G(b_1)$

with a non-null supporting IFS.

Example 3.4

Consider the example 3.1

Let $A = \{a_1, a_2, a_3, a_4\}$ and $B = \{b_1, b_2, b_3\}$. Now let $(F, A)$ and $(G, B_1)$ be two IFSS. Now let $j: (F, A) \rightarrow (G, B_1)$ s.t.,
Since for each element $b \in B$, there exists at least one element $a \in A$ such that $(a, b) \in f$, and $f$ is a surjective IFS-function. Note: In example 3.3 the IFS-function $h$ is injective, but not surjective and in example 3.4 the IFS-function $j$ is both injective and surjective.

**E Definition: Bijective IFS-function**

An IFS-function $f$ from $(A, F)$ to $(B, G)$ is called **bijective** if $f$ is both injective and surjective.

**Example 3.5**

Consider the example 3.1

Let $B = \{b_1, b_2, b_3, b_4\}$ and $C = \{\text{Bank, Research Institute, Hospital, MNC}\}$. Now let $(G, B)$ and $(H, C)$ be two IFSS respectively describing the professions the students may have and placements the students may have are respectively given by,

$$(G, B) = \{G(b_1) = \{(s_1, 0.8), s_2/(0.7), s_3/(0.3), s_4/(0.5), s_5/(0.6), s_6/(0.4)\}, G(b_2) = \{(s_1, 0.4), s_2/(0.6), s_3/(0.5), s_4/(0.2), s_5/(0.7), s_6/(0.3)\}, G(b_3) = \{(s_1, 0.2), s_2/(0.7), s_3/(0.5), s_4/(0.3), s_5/(0.4), s_6/(0.1)\}, G(b_4) = \{(s_1, 0.1), s_2/(0.8), s_3/(0.3), s_4/(0.5), s_5/(0.6), s_6/(0.4)\}\}$$

$$(H, C) = \{H(c_1) = \{(s_1, 0.6), s_2/(0.4), s_3/(0.5), s_4/(0.2), s_5/(0.7), s_6/(0.3)\}, H(c_2) = \{(s_1, 0.2), s_2/(0.7), s_3/(0.5), s_4/(0.1), s_5/(0.6), s_6/(0.2)\}, H(c_3) = \{(s_1, 0.7), s_2/(0.3), s_3/(0.4), s_4/(0.5), s_5/(0.2), s_6/(0.1)\}, H(c_4) = \{(s_1, 0.8), s_2/(0.5), s_3/(0.6), s_4/(0.3), s_5/(0.7), s_6/(0.4)\}\}$$

Now let $(G, B) \to (H, C)$ s.t.,

$g(G(b_1)) = H(c_2) + \text{supporting IFS } \{s_1/(0.7), s_2/(0.6), s_3/(0.5), s_4/(0.6), s_5/(0.3), s_6/(0.4)\}$

$g(G(b_2)) = H(c_4) + \text{supporting IFS } \{s_1/(0.8), s_2/(0.5), s_3/(0.6), s_4/(0.3), s_5/(0.7), s_6/(0.4)\}$

$g(G(b_3)) = H(c_1) + \text{supporting IFS } \{s_1/(0.7), s_2/(0.6), s_3/(0.5), s_4/(0.2), s_5/(0.7), s_6/(0.3)\}$

Since here $g$ is both injective and surjective, so $g$ is a bijective IFS-function.

**F Definition: Constant IFS-function**

An IFS-function $f$ from $(F, A)$ to $(G, B)$ is called a **constant IFS-function** if every element of the domain $(F, A)$ has the same image in $(G, B)$ with non-null supporting IFS.

**G Definition: Identity IFS-function**

An identity IFS-function on an IFSS $(F, A)$ is denoted by $I_{(F,A)}$ and is defined by

$I_{(F,A)}(F(a)) = F(a) \text{ with a non-null supporting IFS } \forall F(a) \in (F, A)$.

**H Definition: Equality of IFS-functions**

Two IFS-functions $f : (F, A) \to (G, B)$ and $g : (F, A) \to (H, C)$ are said to be equal if $f(F(a)) = g(F(a))$ with the same supporting IFS $\forall F(a) \in (F, A)$.

For the equality of two IFS-functions $f$ and $g$ the following conditions must hold:

(i) $f$ and $g$ have the same domain $D$ and

(ii) $\exists x \in D, f(x) = g(x)$ with the same supporting IFS.

**I Definition: Restriction and Extension of an IFS-function**

Let $f : (F, A) \to (G, B)$ be an IFS-function and let $(H, C)$ be a non-empty intuitionistic fuzzy soft subset of $(F, A)$. Then the IFS-function $g : (H, C) \to (G, B)$ defined by

$g(H(c)) = f(H(c)) \text{ with the same supporting IFS } \forall H(c) \in (H, C)$

is said to be the **restriction** of $f$ to $(H, C)$ and in this case, $f$ is said to be an **extension** of $g$ to $(F, A)$.

**J Definition: Composition of IFS-functions**

Two IFS-functions are said to be compatible for
composition if both of them belong to the same universe and the range of one IFS-function be the domain of the other one. Now let \( f : (F, A) \rightarrow (G, B) \) and \( g : (G, B) \rightarrow (H, C) \) be two IFS-functions over the same universe \( U \). Then the composition of \( f \) and \( g \) is denoted by \( g \circ f \) and defined as \( g \circ f : (F, A) \rightarrow (H, C) \) s.t.,

\[
(g \circ f)(a) = g(f(a)) = H(c) \quad \text{with the supporting IFS} \quad I_f = (V_{F(a)} \cap V_{G(b)}) \quad \text{and} \quad I_g = (V_{G(b)} \cap V_{H(c)})
\]

where \( V_{F(a)}, V_{G(b)}, V_{H(c)} \) respectively be the value sets of \( F(a), G(b), H(c) \).

K Definition: Left and Right Inverse of an IFS-function

Let \( f : (F, A) \rightarrow (G, B) \) be an IFS-function. If there exists an IFS-function \( g : (G, B) \rightarrow (F, A) \) s.t.,

\[
g \circ f = I_{(F,A)}
\]

then \( g \) is said to be a left inverse of \( f \). If there exists an IFS-function \( h : (G, B) \rightarrow (F, A) \) s.t.,

\[
f \circ h = I_{(G,B)}
\]

then \( h \) is said to be a right inverse of \( f \).

L Definition: Inverse IFS-function

Let \( f : (F, A) \rightarrow (G, B) \) be an IFS-function. \( f \) is said to be invertible if there exists an IFS-function \( g : (G, B) \rightarrow (F, A) \) s.t.,

\[
g \circ f = I_{(F,A)} \quad \text{and} \quad f \circ g = I_{(G,B)}
\]

In this case \( g \) is said to be an inverse of \( f \).

IV Theorems:

A Theorem:

Let \( f : (F, A) \rightarrow (G, B), g : (G, B) \rightarrow (H, C), h : (H, C) \rightarrow (J, D) \) be three IFS-functions. Then \( h \circ (g \circ f) = (h \circ g) \circ f \)

Proof: Here the composite IFS-functions \( g \circ f, h \circ g \) are defined because range \( f \subseteq \text{domain} \ g \) and range \( g \subseteq \text{domain} \ h \). The composite IFS-functions \( h \circ (g \circ f), (h \circ g) \circ f \) are defined because range \( (g \circ f) \subseteq \text{domain} \ h \) and range \( f \subseteq \text{domain} \ (h \circ g) \).

We shall now prove the equality of the IFS-functions \( h \circ (g \circ f) : (F, A) \rightarrow (J, D) \) and \( (h \circ g) \circ f : (F, A) \rightarrow (J, D) \). Let \( F(a) \in (F, A) \) and let \( f(F(a)) = G(b) \) with supporting IFS \( I_1, g(G(b)) = H(c) \) with supporting IFS \( I_2 \),

\[
h(H(c)) = J(d) \quad \text{with the supporting IFS} \quad I_3,
\]

\[
I_1 = V_{F(a)} \cap V_{G(b)}, I_2 = V_{G(b)} \cap V_{H(c)}, \quad \text{respectively}
\]

where \( I_3 = V_{H(c)} \cap V_{J(d)} \). Let \( V_{F(a)}, V_{G(b)}, V_{H(c)} \) be the value sets associated with the predicates \( F(a), G(b), H(c) \). Then

\[
(g \circ f)(a) = g(f(a)) = g(G(b)) \quad \text{with supporting IFS} \quad I_1,
\]

\[
h((H(c)) = h(g(G(b))) = h((H(c))) \quad \text{with supporting IFS} \quad I_2
\]

\[
(J(d)) = J(d) \quad \text{with the supporting IFS} \quad I_3
\]

Therefore

\[
h \circ (g \circ f)(a) = (h \circ g)(G(b)) \quad \text{with supporting IFS} \quad I_1
\]

\[
= h((H(c))) \quad \text{with supporting IFS} \quad I_2
\]

\[
(J(d)) = J(d) \quad \text{with the supporting IFS} \quad I_3
\]

Therefore

\[
h \circ (g \circ f)(a) = (h \circ g)(G(b)) \quad \text{with supporting IFS} \quad I_1
\]

\[
(J(d)) = J(d) \quad \text{with the supporting IFS} \quad I_3
\]

B Theorem:

If \( f : (F, A) \rightarrow (G, B) \) and \( g : (G, B) \rightarrow (H, C) \) be both injective IFS-functions then the composite IFS-function \( g \circ f : (F, A) \rightarrow (H, C) \) will be injective provided \( f \) maps different elements of \( (F, A) \) to different elements with different predicate parts of \( (G, B) \).

Proof:

Let \( F(a_1), F(a_2) \) be two distinct elements of \( (F, A) \).

\[
f(F(a_1)) = G(b_1) \quad \text{with supporting IFS} \quad I_1,
\]

\[
f(F(a_2)) = G(b_2) \quad \text{with supporting IFS} \quad I_2
\]

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where 
\[ I_1 = V_{F(a_1)} \cap V_{G(b_1)}, I_2 = V_{F(a_2)} \cap V_{G(b_2)}; \]
respectively be the value sets associated with the predicates 
\( F(a_1), G(b_1), F(a_2), G(b_2). \) Since \( f \) is injective, then \( G(b_1), G(b_2) \) are distinct elements of \( (G, B). \)

Case-(1) \( G(b_1), G(b_2) \) are distinct elements of \( (G, B). \)

Let 
\[ g(G(b_1)) = H(c_1) \text{ with supporting IFS } I_1, \]
\[ g(G(b_2)) = H(c_2) \text{ with supporting IFS } I_4, \]
where 
\[ I_3 = V_{G(b_1)} \cap V_{H(c_1)}, I_4 = V_{G(b_2)} \cap V_{H(c_2)} \]
respectively be the value sets associated with the predicates 
\( G(b_1), H(c_1), G(b_2), H(c_2). \) Since \( g \) is injective, either \( H(c_1), H(c_2) \) are distinct elements of \( (H, C) \) or, \( I_3 \neq I_4 . \)

Case-(1.1) \( H(c_1), H(c_2) \) are distinct elements of \( (H, C). \)

Now 
\[ (g \circ f)(F(a_1)) = g(f(F(a_1))) = g(G(b_1)) \text{ with supporting IFS } I_1 \]
\[ = H(c_1) \text{ with supporting IFS } I_3 \]
\[ (g \circ f)(F(a_2)) = g(f(F(a_2))) = g(G(b_2)) \text{ with supporting IFS } I_4 \]
\[ = H(c_2) \text{ with supporting IFS } I_4 \]

Therefore 
\[ F(a_1) \neq F(a_2) \Rightarrow (g \circ f)(F(a_1)) \neq (g \circ f)(F(a_2)) \]
[Since \( H(c_1) \neq H(c_2) \) ]
Hence \( g \circ f \) is injective.

Case-(1.2) \( H(c_1) = H(c_2) \) (say) and \( I_3 \neq I_4 . \) Now 
\[ (g \circ f)(F(a_1)) = g(f(F(a_1))) = g(G(b_1)) \text{ with supporting IFS } I_1 \]
\[ = H(c) \text{ with supporting IFS } I_3 \]
\[ (g \circ f)(F(a_2)) = g(f(F(a_2))) = g(G(b_2)) \text{ with supporting IFS } I_2 \]
\[ = H(c) \text{ with supporting IFS } I_4 \]
Therefore 
\[ F(a_1) \neq F(a_2) \Rightarrow (g \circ f)(F(a_1)) \neq (g \circ f)(F(a_2))[ \]
Since \( I_3 \neq I_4 \) ]
Hence \( g \circ f \) is injective.

Case-(2)
\[ G(b_1) = G(b_2) \text{ and } I_1 \neq I_2. \]
Let
\[ g(G(b_1)) = g(G(b_2)) = H(\overline{c}) \text{ with supporting IFS } I. \]

Now 
\[ (g \circ f)(F(a_1)) = g(f(F(a_1))) = g(G(b_1)) \text{ with supporting IFS } I_1 \]
\[ = H(\overline{c}) \text{ with supporting IFS } I \]
\[ (g \circ f)(F(a_2)) = g(f(F(a_2))) = g(G(b_2)) \text{ with supporting IFS } I_2 \]
\[ = H(\overline{c}) \text{ with supporting IFS } I \]
Therefore
\[ F(a_1) \neq F(a_2) \Rightarrow (g \circ f)(F(a_1)) \neq (g \circ f)(F(a_2)) \]
Hence \( g \circ f \) is not injective.

Thus \( g \circ f \) is injective provided \( f \) maps different elements of \( (F, A) \) to different elements with different predicate parts of \( (G, B). \)

Note: The converse of the theorem \( B \) is not true. However if \( g \circ f \) is injective then \( f \) is injective (while \( g \) need not be).

C Theorem:
If \( f : (F, A) \rightarrow (G, B) \) and \( g : (G, B) \rightarrow (H, C) \) be two IFS-functions such that the composite IFS-function \( g \circ f : (F, A) \rightarrow (H, C) \) is injective then \( f \) is injective.

Proof: If possible, let \( f \) be not injective. Then there exist two distinct elements \( F(a_1), F(a_2) \) in \( (F, A) \) such that \( f(F(a_1)) = G(b) \) and \( f(F(a_2)) = G(b) \) with the same supporting IFS \( I_f \)
where
\[ I_f = V_{F(a_1)} \cap V_{G(b)} = V_{F(a_2)} \cap V_{G(b)}; V_{F(a_1)} \cap V_{G(b)}; V_{F(a_2)} \cap V_{G(b)} \]
respectively be the value sets associated with the predicates \( F(a_1), G(b), F(a_2). \)
Now
\[ g(f(F(a_1))) = g(G(b)) \text{ with the supporting IFS } I_g \]
\[ = H(c) \text{ with the supporting IFS } I_g \text{ (say) } \]
where \( I_g = V_{G(b)} \cap V_{H(c)}; V_{G(b)} \cap V_{H(c)} \) respectively be the value sets associated with the predicates \( G(b), H(c). \) Then
\[ g(f(F(a_2))) = g(G(b)) \text{ with the supporting IFS } I_f \]
\[ = H(c) \text{ with the supporting IFS } I_g \]
So \( g(f(F(a_1))) = g(f(F(a_2))), \) i.e.,
\( (g \circ f)(F(a_1)) = (g \circ f)(F(a_2)) \) and this contradicts that \( g \circ f \) is injective.
Therefore \( f \) is injective. (proved)

**D Theorem:**
If \( f : (F, A) \to (G, B) \) and \( g : (G, B) \to (H, C) \) be both surjective IFS-functions then the composite IFS-function \( g \circ f : (F, A) \to (H, C) \) is surjective.

**Proof:** Let \( H(c) \) be an element of \( (H, C) \). Since \( g \) is surjective, there is at least one pre-image of \( H(c) \) in \( (G, B) \). Let one such be \( G(b) \). Then \( G(b) \in (G, B) \) and \( g(G(b)) = H(c) \) with the supporting IFS \( I_g \)
where \( I_g = V_{G(b)} \cap V_{H(c)} \).
Since \( f \) is surjective and \( G(b) \in (G, B) \), there is at least one pre-image of \( G(b) \) in \( (F, A) \). Let one such be \( F(a) \).
Then \( f(F(a)) = G(b) \) with the supporting IFS \( I_f \)
where \( I_f = V_{F(a)} \cap V_{G(b)} \).
\( (g \circ f)(F(a)) = g(f(F(a))) \)
\[ = g(G(b)) \text{ with the supporting IFS } I_f \]
\[ = H(c) \text{ with the supporting IFS } I_g \]
This implies that \( H(c) \) has a pre-image in \( (F, A) \) under the IFS-function \( g \circ f \).
Since \( H(c) \) is arbitrary, \( g \circ f \) is surjective. (proved)

**Note:** In order that \( g \circ f \) may be surjective it is not necessary that \( f \) is surjective.

**D Theorem:**
If \( f : (F, A) \to (G, B) \) and \( g : (G, B) \to (H, C) \) be both bijective IFS-functions and \( f \) maps different elements of \( (F, A) \) to different elements with different predicate parts of \( (G, B) \) then the composite IFS-function \( g \circ f : (F, A) \to (H, C) \) is bijective.

**Proof:** This is a combination of the theorems \( B \) and \( D \).
**Note:** The converse of the theorem is not true. However if \( g \circ f \) is bijective then \( f \) is injective and \( g \) is surjective.

## V Change of The Climate System

Drastic change of the climate system in the global warming scenario is a major problem which is affecting people and the environment continuously. The climate system mainly consists of five components: atmosphere, hydrosphere, cryosphere, land surface and biosphere. Observation regarding \((i)\) the increase in average temperature in air and ocean, \((ii)\) widespread melting of ice and \((iii)\) rising global average sea level, clearly states the warming of the climate system. In the following problem we are considering this relevant topic.

**Problem:** There are different types of human activities like increasing concentration of greenhouse gases(GHG) in the atmosphere, deforestation, fossil fuels emission, land use changes(from natural to urban) etc for which global climate change is occurring continuously. In this study we are considering the main two human activities: \((i)\) increasing concentration of greenhouse gases(GHG) in the atmosphere and \((ii)\) deforestation which causes harmful effect on the climate system. Now the role of human activities may vary and their degree of impact on the climate change depending on the place concerned. So considering this variety we choose three types of countries: the countries with a huge industrial belt \((C_1)\), the countries with a dense population \((C_2)\) and the countries with little industry and population \((C_3)\) such as the countries in the polar region which are suffering from the climate change such as increase in average temperature in air, melting of ice and occurrence of flood etc due to increasing concentration of GHG in the atmosphere and deforestation. Nowadays many social welfare organizations have more active in their battle against pollution. They are undertaken programs in order to create awareness about the ill effects of this drastic climate change and their long standing impact on human civilization. Now in this study we have collected data from two sources: an official record and a record of a rescue operation team. From the official record, we have that per decade the concentration of GHG in the atmosphere increases at the rate 0.8% (approx) in the industrial belt, 0.6% (approx) in the dense populated area and 0.5% (approx) in the polar region. Moreover the same source reveals that, per decade, deforestation occurs at the rate 0.7% (approx) in the industrial belt, 0.8% (approx) in the dense populated area and 0.2% (approx) in the polar region. Now according to the rescue operation team record in the industrial belt, dense populated area, polar region the concentration of GHG in the atmosphere does not increases at the rate 0.1%,0.2%,0.5% (approx) and deforestation does not occur.
0.1%, 0.1%, 0.5% (approx) per decade respectively. Similarly we have from official record, per decade, the average temperature of air increases at the same rate that of the rate of increasing concentration of GHG in the atmosphere whereas the rescue operation team record it does not occur in the respective regions at the rate 0.2%, 0.3%, 0.4% (approx) per decade. Now from the official record per decade, melting of ice and occurrence of flood happen respectively at the rate 0.1%, 0.5% (approx) in the industrial belt, 0.2%, 0.3% (approx) in the dense populated area and 0.8%, 0.7% (approx) in the polar region. Again from the rescue operation team record, per decade, melting of ice and occurrence of flood do not happen respectively at the rate 0.7%, 0.3% (approx) in the industrial belt, 0.5%, 0.5% (approx) in the dense populated area and 0.1%, 0.2% (approx) in the polar region. Moreover from the official record massive and low harmful effects occur on the climate of the countries C1, C2, C3 at the rate 0.8%, 0.6%, 0.5% (approx) and 0.2%, 0.3%, 0.4% (approx) and from the rescue operation team record they do not occur at the rate 0.1%, 0.2%, 0.4% (approx) and 0.7%, 0.5%, 0.5% (approx) per decade respectively. Now from both of these sources we can conclude that average temperature in air increases mainly due to increasing concentration of GHG in the atmosphere and flood occurs mainly due to deforestation. We also have from both of these sources that increase in average temperature in air, melting of ice and occurrence of flood are responsible for massive harmful effect. Now the problem is to find out the rate of harmful effects on climate due to human activities.

To solve this problem we have to construct an IFS-function which describes the rate of harmful effects on climate due to human activities. So considering the given information we have to form an IFS-function from an IFSS (describing harmful human activities in the countries,) to another IFSS (describing the rate of harmful effects on the climate of the countries). For this purpose first we have to apply the concepts of intuitionistic fuzzy soft set, intuitionistic fuzzy soft function and then use the property of a constant IFS-function together with the concept of composition of IFS-functions.

For solving these the following notations are used:

- \{ increasing concentration of GHG in the atmosphere, deforestation \} = \{a_1, a_2\},
- \{ increase in average temperature in air, melting of ice, occurrence of flood\} = \{b_1, b_2, b_3\}

\{not massive, not low\} = \{c_1, c_2\}

Therefore, in the parlance of Intuitionistic Fuzzy Soft Set, the finite universe, \( U = \{C_1, C_2, C_3\} \) and the sets of parameters, \( A = \{a_1, a_2\}, B = \{b_1, b_2, b_3\}, C = \{c_1, c_2\} \). Now from the given data, according to the definition of intuitionistic fuzzy soft set, we have the intuitionistic fuzzy soft sets \((F, A), (G, B)\) and \((H, C)\) respectively describing harmful human activities in the countries, signs of climate change in the countries, the rate of harmful effects on the climate of the countries are respectively defined as

\[ \text{flood do not happen respectively at the rate} \quad 0.7%, 0.3% \quad \text{and} \quad 0.1%, 0.5% \quad \text{in the polar region. Again from the rescue operation}\]
g(G(b_3)) = H(c_1) with supporting

IFS \{C_1/(1.1,7),C_2/(2.5),C_3/(5.4)\}

Now \( f : (F,A) \rightarrow (G,B) \) and \( g : (G,B) \rightarrow (H,C) \) be two IFS-functions over the same universe \( U \) and the domain of \( g \) is the range of \( f \), so \( f \) and \( g \) are compatible for composition as \( g \circ f : (F,A) \rightarrow (H,C) \) s.t.,

\[(g \circ f)(F(a)) = g(f(F(a))) \forall F(a) \in (F,A)\]

Now

\[(g \circ f)(F(a_1)) = g(f(F(a_1))) = g(G(b_1))\]

with supporting IFS \{C_1/(.7,2),C_2/(.6,3),C_3/(.5,5)\}

Hence we have the IFS-function \( g \circ f \) which describes the rate of harmful effects on climate due to human activities by indicating the following information:

**In the industrial belt** \( (C_1) \)

(i) Increasing concentration of GHG in the atmosphere is responsible for massive harmful effect on the climate at the rate 0.7% per decade and is not responsible for massive harmful effect on the climate at the rate 0.2% per decade.

(ii) Deforestation is responsible for massive harmful effect on the climate at the rate 0.5% per decade and is not responsible for massive harmful effect on the climate at the rate 0.3% per decade.

**In the dense populated area** \( (C_2) \)

(i) Increasing concentration of GHG in the atmosphere is responsible for massive harmful effect on the climate at the rate 0.6% per decade and is not responsible for massive harmful effect on the climate at the rate 0.3% per decade.

(ii) Deforestation is responsible for massive harmful effect on the climate at the rate 0.3% per decade and is not responsible for massive harmful effect on the climate at the rate 0.5% per decade.

**In the polar region** \( (C_3) \)

(i) Increasing concentration of GHG in the atmosphere is responsible for massive harmful effect on the climate at the rate 0.5% per decade and is not responsible for massive harmful effect on the climate at the rate 0.4% per decade.

(ii) Deforestation is responsible for massive harmful effect on the climate at the rate 0.5% per decade and is not responsible for massive harmful effect on the climate at the rate 0.4% per decade.

**VI Conclusion:**

In this paper first we have proposed the concept of intuitionistic fuzzy soft function (IFS-function). Here we have defined different types of IFS-function. Moreover we have proven some theorems on these IFS-functions. Finally we have applied IFS-functions and their operations to solve a problem of the Drastic Change in The Climate System.

**References**


